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EVALUATION OF A MARKOV-DECISION MODEL FOR INSTRUCTIONAL
SEQUENCE OPTIMIZATION

Richard D. Wollmer, et al

University of Southern California

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**DEPARTMENT OF PSYCHOLOGY
UNIVERSITY OF SOUTHERN CALIFORNIA**

Technical Report No. 76

**EVALUATION OF A MARKOV-DECISION MODEL FOR
INSTRUCTIONAL SEQUENCE OPTIMIZATION**

October 1975

**Richard D. Wollmer
Nicholas A. Bond**

Sponsored by

**Personnel and Training Research Programs
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SUMMARY

Wollmer's Markov Decision Model for instructional sequence optimization was investigated in a computer-assisted instruction (CAI) context. Two special CAI programs served as vehicles for testing the model; one program (K-Laws) taught the students to solve DC circuit problems using Kirchhoff's Laws; the other (TRIG) gave practice in manipulating the six trigonometric ratios. The K-Laws course had eleven stages or levels; TRIG had five; both courses were arranged in a hierarchical order. The Wollmer model requires that transfer would occur from one stage to the next in a hierarchical learning sequence, and that these effects could be estimated so as to produce an optimal training schedule. To determine the effects of additional practice, half the calibration sample was required to finish one successful trial and half were required to have two successes, before moving on to the next stage. Thirty subjects took the K-Laws course, 80 completed TRIG. Instruction was given at individual CAI terminals.

All subjects finished the course, and learned to perform satisfactorily the final criterion behaviors. Practice effects were unexpectedly slight; people who had one success at each stage of the course had about the same criterion-problem performance as those who had two successes throughout. The average time required to achieve a second success was not appreciably different from that required for the first, and two successes at the immediately preceding level was no better than one, as far as transfer to the next higher stage was concerned. These results indicated that the Wollmer hierarchical model could not improve overall learning much by "optimal" scheduling of practice.

One implication of the findings is that in complex learning hierarchies where the top or most difficult task consists of a collection of previously-learned skills, performance time on that top task may be more dependent on the number of subskills involved than upon the number of practice trials in preceding stages. Another implication is that if practice and transfer effects are to be significant in learning this kind of hierarchically-structured material then a very large number of practice trials may be necessary.

ABSTRACT

Two CAI programs in electronics and trigonometry were written to test the Wollmer Markov Model for optimizing hierarchical learning; calibration samples totalling 110 students completed these programs. Since the model postulated that transfer effects would be a function of amount of practice, half the students were required to complete one practice problem successfully before moving to the next stage; the other half had to do two practice problems successfully.

All students completed the courses satisfactorily. Practice effects were small; students who had one success in each stage did about as well as those who had two successes. The Wollmer model was thus not suitable for optimizing instruction, in terms of minimizing overall time, in these particular courses. Perhaps the main reason for this result was that, as the student works up to the top of the hierarchy, the sheer number of subskills involved in the final task becomes a major determinant of performance time, and number of practice trials has a relatively minor effect, unless a very large number of practice trials is given.

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Thomas D. Carrell wrote the main K-Laws program and ran many of the K-Laws subjects. His work in making the program operational was a most significant part of the project. Don MacGregor was most helpful in writing a portion of the K-Laws parameter estimation routine.

Time-shared computer facilities for running the K-Laws program were provided by Dr. Hovey Reed, Director of the Computer Center at Sacramento State University. We also wish to thank MAJ. GEN. George W. McLaughlin, Commander of Sacramento Air Logistics Center, and to thank COL. Jack C. O'Dell, Commander 2049th Communications Group, and MAJ. William W. Morton, Commander 1833rd Electronics Installation Squadron, for providing technicians as subjects. Throughout the duration of the testing, Capt. Gary Hawksworth, Information Officer at McClellan Air Force Base, was the military liaison. Finally, we should express gratitude to Mr. Barry Hillier, of the McClellan Training Division, who provided office space and phone lines for the terminal.

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I. INTRODUCTION

Research at BTL has, for some years, been concerned with different aspects of instructional technology required for more effective utilization of computers in training and education. Early in this work (Rigney, 1973), an outline was prepared of the elements that constitute an instructional system. This outline is depicted in Figure 1. Each of the elements shown there must be present in some form in a working instructional system. The objective of this laboratory has been to allocate its particular capabilities to research on appropriate elements in this diagram. In those instances where the laboratory has produced and field-tested complete instructional systems, the best available elements were used in those parts of the systems where the laboratory was not, at the time, doing research.

One of the candidates for improvement in instructional systems is the "instructional sequence optimizer," which is shown in the adaptive controller. Atkinson and his colleagues (Atkinson and Paulson, 1973) have convincingly demonstrated the power of certain types of optimization models. The interest of this laboratory in this part of the instructional system relates to technical subject-matter typical of technical training courses in the Navy. It was considered worthwhile to investigate possibilities for developing an instructional sequence optimizer based on operations research techniques. The initial Markov Decision Model was described by Wollmer (1973).

Smallwood (1962) was perhaps the first to propose a definite model; his optimizer assigned that lesson segment which had the highest utility,

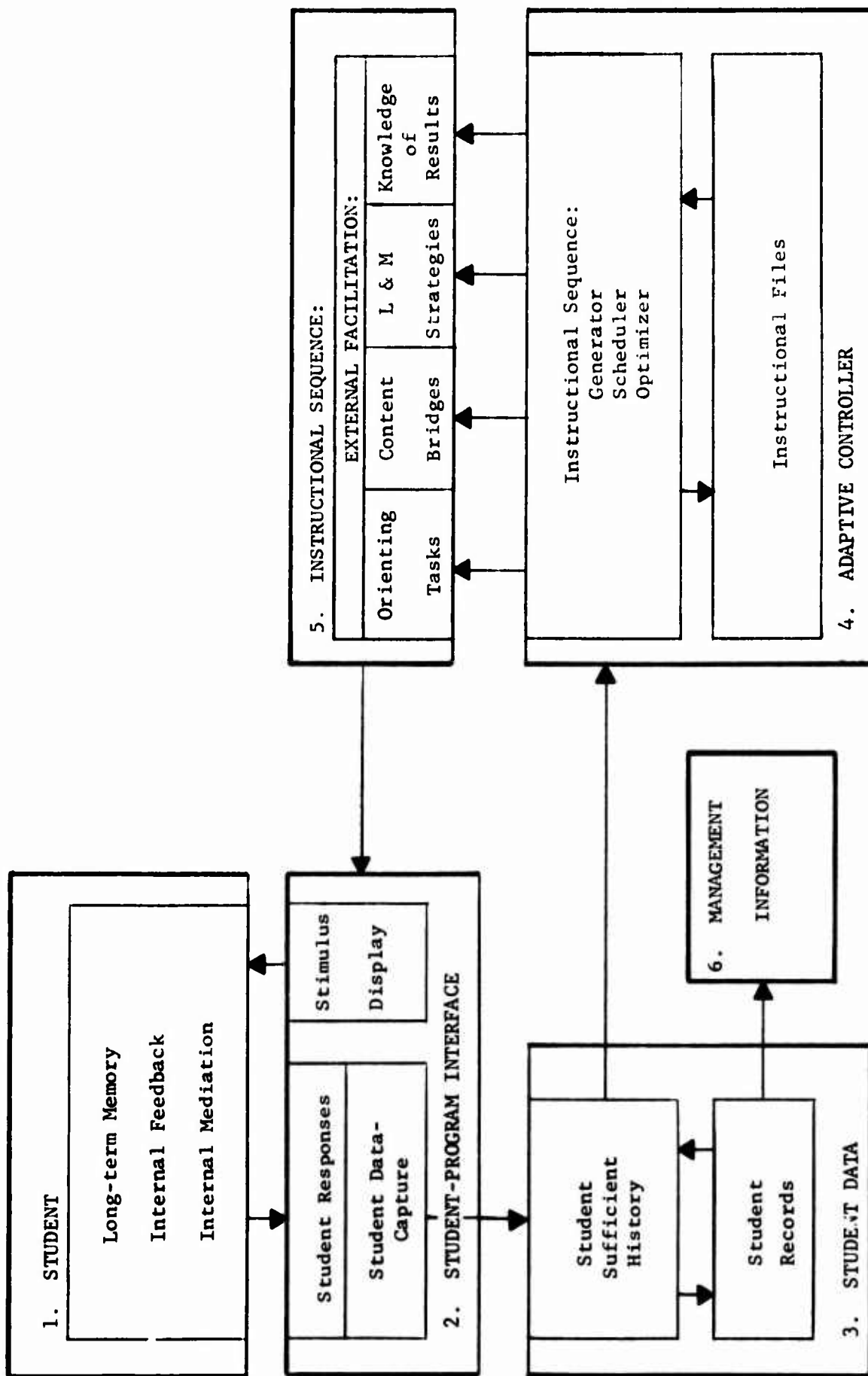


Figure 1 - Outline of Major CAI Subsystems

or highest expected return in the criterion score. Estimates of utility could be made from standardization runs on a sample of subjects. Suppes, Atkinson and their colleagues at Stanford have extended optimization of instruction into several dimensions; for instance, Atkinson & Paulson (1973) define a model which maps learning performance in terms of three aspects: item difficulty, student ability, and learning rate parameters. Using this model, optimization of a vocabulary-learning task was accomplished by estimating the parameters, and giving practice in those items which promised the most gain per practice trial. This optimization was very successful, yielding an efficiency gain on the order of 40 per cent, in terms of time saved. Chant and Atkinson (1973) developed an optimization technique for allocating instructional effort to two interrelated strands of learning material. Their key assumption was that the learning rate for each of the two strands depends solely on the difference between the achievement levels on the two strands.

The Wollmer (1973) model assumes that the course being taught proceeds in a definite hierarchical nature. Thus if levels are numbered consecutively with 1 being the most difficult, and the highest numbered being the easiest, mastering of the material at a particular level implies mastering of the material at all higher numbered levels. Furthermore, it is assumed that successful completion of a problem at one level increases the probability of being able to successfully complete a problem at the next most difficult level, following instruction at that level. This model can be considered a special case of a partially observable Markov decision process over an infinite planning horizon. Smallwood and Sondik (1973) formulate and solve such a decision process over a finite planning

horizon. The principal purpose of this study was to provide data for parameters estimation in this model.

Research in computational techniques for the more general infinite horizon Markov decision process is currently being done at BTL and results will be reported in future publications. These results will not only offer an alternative computational technique for the model described above but also will allow one to relax some of its more restrictive assumptions.

II. THE TEACHING PROGRAMS

Two teaching programs were specially written as vehicles for testing the Wollmer hierarchical model. One of these programs (K-Laws) taught the student to solve DC circuit problems using Kirchhoff's voltage and current laws; the other (TRIG) provided instruction and practice in manipulating the six trigonometric ratios.

The Kirchhoff's Laws Course

The Kirchhoff's and Ohm's relations are among the most-taught principles of science. All students in electronics and physics are supposed to master them, and of course, many textbooks and courses feature these principles throughout. Even so, there is plenty of evidence that simple circuit analysis remains difficult for many technicians and students. A real problem, apparently, is the designation or translation of physical circuit quantities into the Kirchhoff and Ohm equations. Solving DC circuit problems via these equations is analogous to working out a "word problem" in algebra: once the equations are set up, everything can proceed smoothly; the difficulty is to translate the verbal statements and conditions into the algebraic framework.

The K-Laws course was organized hierarchically, with the desired criterion skill at the end of the course being a demonstrated ability to calculate certain voltage drops in a three-wire circuit like that shown in Fig. 2. For a typical problem near the end of the course, the student would be shown the schematic in Fig. 2, with the following parameters:

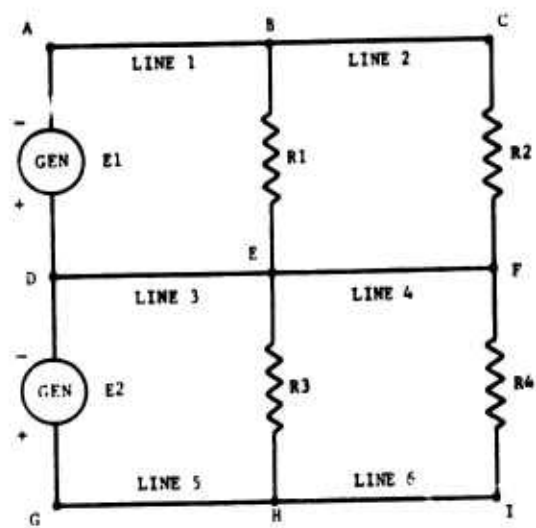


Figure 2. Schematic of three-wire circuit

R_1 draws 16 amps
 R_2 draws 23 amps
 R_3 draws 13 amps
 R_4 draws 9 amps
Generator 1 is delivering 114 volts, polarity as shown
Generator 2 is delivering 109 volts, polarity as shown
Each wire, A, B, C, D, E, F, has a resistance of 0.5 ohms.

He would then be asked, what is the voltage drop across R_4 ?

Such circuit problems cannot be solved through guessing. Several calculations are necessary, along with careful definition of the relations that prevail in the circuit. There are various ways to find the desired answer. For most technicians, an effective method is to determine the amount and direction of all the currents, to convert the various current loads into voltage drops by multiplying resistances and currents, and finally to set up Kirchoff's voltage law in one unknown and solve for the missing voltage.

The requisite skills to accomplish this final criterion performance are laid out in Fig. 3, which displays a presumed "learning hierarchy" for the college-level subjects that were used (Gagne, 1970). For this sample of people, certain algebraic and verbal skills were assumed; if the same criterion skills had to be imparted to seventh-graders, then the hierarchy would be considerably extended.

There are two major paths in the K-Laws learning hierarchy. On the left is what might be called the "voltage drop" sequence; here the student learns or reviews the Ohm's law formulas and practices using them in several circuits; he also applies the "sign rule" regarding the direction of current flow and the sign of the voltage drop through a resistance. At the right side of the hierarchy, there is a chain of subskills involving the determination of current direction and quantity in a three-wire, two generator circuit with several passive loads. Here the information to be

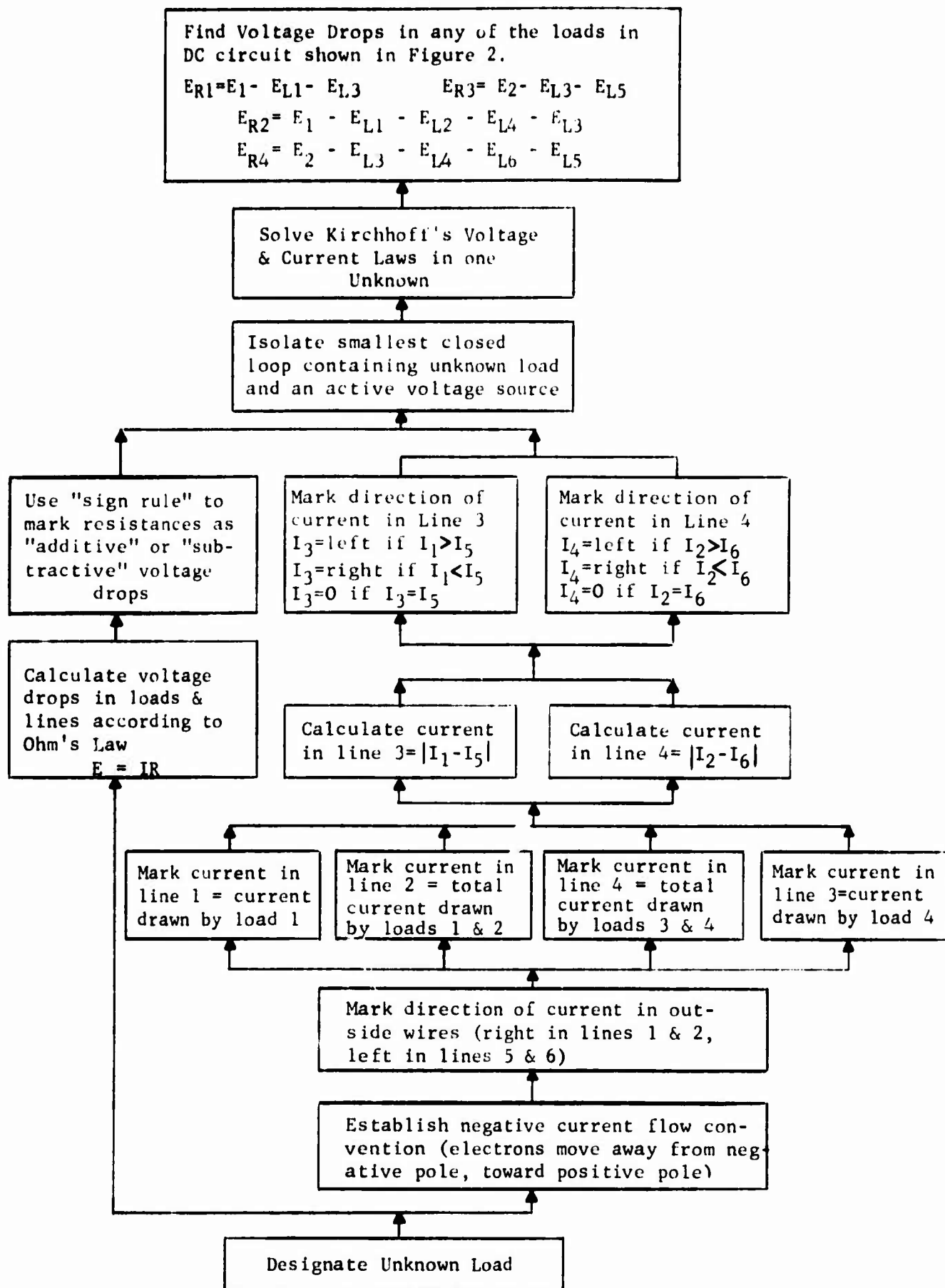


Figure 3 - A Gagne' hierarchy for the K-Laws problem solving procedures.

taught is, perhaps, less general than the voltage-drop material; the solution sequence is confined to a particular three-wire configuration, and might not hold exactly for similar but different circuit layouts. The voltage drop principles, in contrast, are extremely broad in application. Both chains of the hierarchy have to be mastered to solve the final criterion problem.

If a learning hierarchy is valid, then substantial and positive transfer from "lower" to "higher" stages should obtain; that is, those who can perform the subskills "underneath" a final behavior should be more likely to succeed in the final task, than those who cannot accomplish the subskills. Gagne and his associates (1962) demonstrated such transfer effects in a mathematics task with school children. For the K-Laws program, it was assumed that it would be a good instructional strategy to require every subject to demonstrate a definite capability at every sub-skill level, before advancing to the next part of the course.

Eleven "levels" or course stages were defined; those units corresponded roughly to tasks in the learning hierarchy. Levels were numbered so that high numbers represents easy or early parts of the course. Some levels were very elementary and easy, such as the lesson involving direction of electron flow out of a battery. The last two or three stages, though, were quite involved, since the student was then applying several newly-learned skills from all or most of the preceding stages, and he was usually performing these skills in a definite order.

To the student, a standard teletype terminal was the major piece of teaching hardware in the set up. This terminal was, of course, driven by a time-shared teaching program, which was written in the BASIC language

and was stored in a distant central computer. A random access slide projector was used to display circuit diagrams on the wall, in front of the student. Two booklets were also part of the teaching package; one contained lesson material and illustrations, the other had blank circuit diagrams for the student to use as he practiced some of the lessons. A small hand-held calculator was provided for calculations; each student worked a few problems on it before beginning the course.

When a student appeared for instruction and first logged on to the teletype, the program asked him eleven questions regarding his knowledge of electronics. These questions related directly to the eleven stages of K-Laws course content. In fact, question number 11 essentially asked the student if he knew level 11, question 10 if he could perform the criterion tasks in level 10, etc. If he answered "yes" to a question, then he was given a sample problem to determine if that yes answer was valid. For example, at level 6, question 6 was: Can you calculate the resistances in a parallel DC circuit?" If a student typed a "yes" to this question, then a single test item was given to him, to see if he actually could perform. The program generated a circuit of three or four resistors in parallel, and asked the student to figure out the total resistance across them. Comparison of the student's answer to the correct one was immediately performed by the program and printed out for the student to see. The entering skills test yielded, then, a series of eleven "yes" or "no" answers, along with a pre-test right-wrong score for each "yes" item.

Once the student began the main K-Laws course, he worked through the program at his own pace, with a research staff member standing nearby to handle such matters as computer shutdowns, log-outs, restarts, and timer resets. The staff member did not supplement the lesson materials or attempt

to explain difficult items. When learning difficulties did occur, and the student was perplexed, he was told to go back over the lesson material carefully, in step-by-step fashion. When a subject finished studying the material in a given teaching unit, he typed "D" (for "Done") on the keyboard. The program then made up practice problems for that teaching unit, with some remediation loops automatically keyed to errors.

Generally, the system worked satisfactorily. On one or two occasions, the problem-generator in the master program happened to produce degenerate or "insoluble" problems for the student. Some of the data were obtained via time-sharing with a computer just half a mile across campus; the rest of the data came by operating through a time-sharing center some 400 miles away from the teaching terminal. For this distant operation, noisy telephone lines caused total shutdown a few times. As it turned out, a few students elected not to use the slide projector to display the circuit diagrams; they referred solely to the booklets for that information.

After each problem answer was received by the computer terminal, the system immediately printed out a "correct" or "incorrect" evaluation of the answer provided by the student, furnished the correct answer, and indicated the time that the student had spent on that problem. If an answer was wrong, the student had to keep working in that same level, until either one correct or two correct answers were achieved. Whether the student received one practice problem or two problems was decided by a coin flip, when he logged onto the system.

Thirty subjects completed the K-Laws course. Fifteen of them were college students; fifteen were military technicians who were working in electronics or related fields at McClellan Air Force Base, California.

College students were paid a nominal hourly fee for participating; the military people were ordered to appear, and their only recompense was time off from regular duties. The subjects differed markedly in their familiarity with electronics concepts. Several of the college subjects were engineering majors and had completed one or more electricity courses; such students might claim that they already knew much of the material in the course, but no student could solve the pretest problems in the last three (most difficult) stages without some practice at the terminal. At the other extreme were some liberal-arts majors who had almost no technical experience with voltage drops and circuit diagrams; some of these subjects said that they "weren't very good at this sort of thing," but all of them persisted and solved the criterion problems at the end. Breaks were given about every one-and-a-half to two hours during the teaching. In most cases the program was completed in a single day of six to eight hours; about a third of the subjects had to appear on two or more days because of personal scheduling difficulties, system breakdowns, and the like.

The Trigonometry Course

A short course in trigonometry (TRIG) served as the second vehicle to test the model. The TRIG course consisted of five levels, and as in the K-Laws program, they were arranged in a strict hierarchical structure. The levels were numbered so that level five indicated the easiest or entering lesson, and level one represented the most difficult. In order for a student to know the material at a given level (say Level 2), he also had to use significant parts of the material at all higher numbered levels (say Levels 5 through 3).

As a start, in Level 5, the student was given the definition of the six basic trigonometric ratios--the sine, cosine, tangent, cotangent, secant, and cosecant. Then he was presented with a right triangle which had the side lengths displayed, and was asked to find the six basic ratios for that triangle. Level 4 treated the cofunction relationships; in this unit the student learned that the cosine, cotangent, and cosecant of an angle are equal to the sine, tangent, and secant of the complementary angle. Level 3 instruction used the relation $\sin^2\theta + \cos^2\theta = 1$, and gave practice in working out values from this equation. Level 2 taught how all the trig ratios can be computed from either the sine, cosine, secant, or cosecant. Finally, in Level 1, the most advanced unit, the student was shown how to determine all six ratios from either the tangent or cotangent. Then he was given either the tangent or cotangent of an angle, and asked to find the other five basic trigonometric ratios. Satisfactory performance in this last teaching unit resulted in "graduation."

As in the K-Laws sequence, the student was asked questions about the material in a pretest session, before he began the instruction. Thus, one question, keyed to Level 3, asked: "Do you know how to compute the cosine of an angle from its sine?" There were five such preliminary questions, one for each level.

After the student answered these five questions he was given instructions and problems at levels five, four, three, two and one in that order. A student advanced from one level to another by successfully solving either one or two problems at that level, the number for each man being determined by a random number generator. In order for a student to gain credit for a problem, he had to do all parts correctly. Thus if a student

received a problem at level five and gave an incorrect answer for one of the six trigonometric ratios, he was immediately informed that he missed that problem and then presented with a new triangle, and was asked to solve for a new series of six ratios. Before being given a new problem the student always had the option of reviewing instruction.

Eighty TRIG subjects were run on a PLATO IV terminal. Subjects were psychology students who received "subject pool" course credit for participating; each one was scheduled for two hours at the terminal, but most did not require that long to finish the course. TRIG was written in the TUTOR language.

III. PARAMETER ESTIMATION

Two probability vectors are needed by the Wollmer (1973) model. The components of these vectors are:

$$p_i = P \left[\begin{array}{l} \text{student can initially solve a problem successfully at} \\ \text{level } i \end{array} \right]$$

$$q_i = P \left[\begin{array}{l} \text{student can perform at level } i-1 / \text{ student solves a problem} \\ \text{correctly at level } i \text{ and could not perform successfully at} \\ \text{level } i \text{ before.} \end{array} \right]$$

A large group of subjects were run, as described in Section II, to collect data that could be used for estimating initial values of these parameters.

According to the model, the probability that a student can perform successfully at level i after solving k problems successfully at level $i+1$ is $1 - (1 - p_i)(1 - q_{i+1})^k$. Let \bar{X}_{1i} be the proportion of incorrect solutions at level i by students who solved one problem correctly at level $i+1$, and let \bar{X}_{2i} be the proportion of incorrect solutions at level i by students who solved two problems correctly at level $i+1$. Then \bar{X}_{1i} and \bar{X}_{2i} are estimators of quantities as follows:

$$\bar{X}_{1i} = (1 - \hat{p}_i)(1 - \hat{q}_{i+1}) \quad (1)$$

$$\bar{X}_{2i} = (1 - \hat{p}_i)(1 - \hat{q}_{i+1})^2 \quad (2)$$

This is, \bar{X}_{1i} and \bar{X}_{2i} estimate the probability of failure at level i by students who solved one and two problems correctly respectively at level $i+1$. Solving these for p_i and q_{i+1} one obtains:

$$\hat{p}_i = 1 - \bar{X}_{1i}^2 / \bar{X}_{2i} \quad (3)$$

$$\hat{q}_{i+1} = 1 - \bar{X}_{2i} / \bar{X}_{1i} \quad (4)$$

The quantities \bar{X}_{1i} and \bar{X}_{2i} may depend on the student's answers to the pretest questions. Thus in a five-level teaching system such as TRIG, if \hat{p}_i and \hat{q}_{i+1} are to be estimated solely on the basis of the answer to pretest question j , one would obtain $\bar{X}_{1i} = Y_{ij}/(W_{ij} + Y_{ij})$ and $\bar{X}_{2i} = Y_{ij+6}/(W_{ij+6} + Y_{ij+6})$ if the student answered yes to pretest question j ; $\bar{X}_{1i} = Z_{ij}/(\bar{X}_{ij} + Z_{ij})$ and $\bar{X}_{2i} = Z_{ij+6}/(X_{ij+6} + Z_{ij+6})$ if the student answered no to pretest question j . (Full details, definitions of W , X , Y , and Z , and data matrices are given in Appendix 1.)

Since the student answered either yes or no to five (or eleven) different pretest questions, there were five (or eleven) possible estimates of \bar{X}_{1i} and \bar{X}_{2i} and consequently of \hat{p}_i and \hat{q}_{i+1} . Each of these estimates for \bar{X}_{1i} and \bar{X}_{2i} were obtained from the control group and are displayed in Tables 9 through 12. There are several ways of estimating the p and q vectors from this data, if the model is used to guide students through the course. For example, for TRIG (K-Laws) one might average estimates of \bar{X}_{1i} and \bar{X}_{2i} and use this to solve for \hat{p}_i and \hat{q}_{i+1} in (3) and (4). Another method is to obtain five possible estimates of \hat{p}_i and \hat{q}_{i+1} by substituting the pairs of estimates of \bar{X}_{1i} and \bar{X}_{2i} , and then to average these. A third possibility is to base the estimates of \hat{p}_i and \hat{q}_{i+1} solely on the basis of the answer to i which is the one specifically directed at level i . Still another way is to let a yes answer to a question be considered a one score, to count a no answer as zero score, and then let \hat{p}_i and \hat{q}_{i+1} be a linear combination of the scores on the pretest questions.

First, however, it was necessary to run initial samples through the program to provide "calibration" data.

Two samples of students were run through all levels of each course, under either a two or one success per-level policy. Allocation of policies was by random assignment to students at entry in the K-Laws course, and by random assignment to students at each successive level in the TRIG course. Answers to pretest questions were tabulated, but were not used for weights nor for entry-level decisions (all students took all levels), in these initial, "calibration" samples.

IV. RESULTS: CALIBRATION DATA

The questions of central importance to evaluating the usefulness of this type of model are (1) whether the policy of requiring two successes at a level resulted in better performance at the next level than the one-success-at-a-level policy, and (2) whether learning occurred within a level, as indicated by comparing first success with second success data. The data in Tables 1 and 2, from the K-Laws course, bear on these questions.

Table 1

Overall Failure Rates: (1) For One Success Within a Level and (2) for Two Successes Within a Level; K-Laws Course

Levels	One Success Policy (N = 15)	Two Success Policy (N = 15)	
	1	1st	2nd
11	.079	.071	.071
10	.079	.133	.000
9	.143	.071	.133
8	.133	.278	.235
7	.278	.294	.250
6	.235	.235	.133
5	.435	.519	.278
4	.308	.315	.167
3	.400	.593	.389
2	.538	.091	.333
1	.500	.556	.385
Mean	.284	.287	.216
SD	.166	.195	.127

t (within levels) = 1.716 p(10df) = .12

t (between levels) = .033 p(20df) = .97

Although there was a slight reduction in probability of failure for the second success within a level, from .28 to .22, this was not a statistically significant difference. This suggests that the policy of requiring two successes per level was too "lenient;" that is, not enough extra practice was required to differentiate between policies. Comparison of failure rate means for the first success at a level, column 1, also reveals practically no difference (.284 and .287) between the effects of the two policies. Thus, requiring students to succeed twice within a level did not reduce their failure rate for the first success at the next level. This overall failure rate was the same as that for students who were required to succeed only once at a level.

Examination of the time data for the same conditions (Table 2), reveals a similar story.

Table 2

Mean Time (Minutes) per Problem: (1) for
One Success within a Level and (2) for
Two Successes within a Level; K-Laws Course

Levels	One Success Policy	Two Success Policy	
	1	1st	2nd
11	.80	.97	1.0
10	.88	.86	1.93
9	.93	.97	1.25
8	3.53	4.00	2.63
7	3.76	2.16	2.33
6	1.73	3.60	2.77
5	8.18	7.42	5.83
4	8.45	6.87	6.07
3	7.66	7.58	4.88
2	4.26	4.61	4.83
1	9.88	7.68	8.23
Mean	4.55	4.25	3.80
SD	3.43	2.79	2.31

t (within levels) = 1.347 p (10df) = .21
t (between levels) = .228 p (20df) = .82

In Table 2, comparison between the overall means (4.55 vs 4.25) for the first success within a level, for the two policies, obviously yielded no significant difference. While there was a slight overall decrease in mean time to achieve the second success within a level in comparison to mean time to achieve the first, this difference was neither statistically nor practically significant. Further, the two-success policy did not have a cumulative effect between levels. Otherwise, it should have resulted in a smaller overall mean time to achieve the first success in a level than did the one-success policy. Other data from the calibration samples are summarized in Appendix 1.

V. DISCUSSION OF RESULTS.

It is clear from the comparisons in Tables 1 and 2 that different amounts of practice on the same problems (within a course level) had only slight effects on probability of failure on subsequent problems of the same kind, and did not positively influence performance at the next level. Since the Wollmer Markov decision model requires that this positive influence occur, the model apparently cannot be applied to the kind of course material that was used here. It also is clear that the extra practice the student received under the two-success policy did not have an appreciable effect on mean time to successful solution of a problem at the next level. Again, the model requires that the effects of practice transfer across levels. The problems used in the K-Laws and TRIG programs were relatively complicated, in that each problem consisted of several parts, and required that the students perform a series of operations, often in a certain sequence. Under these circumstances, time to perform should be determined by the number of operations to be performed, until the practice-for-fluency stage is reached, at which "chunking" of operations can occur; this might reduce the correlation between time to perform and number of operations to be performed. If this phenomenon occurs, it is likely to occur only after long periods of overlearning, entailing intensive practice, indeed.

In a radar-intercept trainer (Rigney, et al., 1974), students performed the same six mental arithmetic problems over and over. Over a series of 100 to 150 practice problems, in 10 to 20 sessions, involving 10 to 15 hours of practice, overall mean latency to do all six problems was

reduced from 68 seconds to 29 seconds, or, by a factor of 2.3. It should be noted also that this was a real-time situation, in which students were driven to perform faster by the requirement to keep up with a developing tactical problem. The present study did not impose this degree of time pressure on the learner.

The different levels in the K-Laws and TRIG courses were created by introducing new rules, or procedures. While the student needed to use virtually all of the rules or procedures he had learned at preceding levels, it still seems likely that the new elements in problems at each succeeding level were sufficiently novel to reduce inter-level transfer effects.

To see if a much larger amount of practice would affect time to perform, three additional K-Laws subjects were run under the condition that five correct solutions were required at each of the eleven levels before "graduation" from a stage of instruction. This policy did seem to promote more learning and transfer; the mean time for problem solution in the final stage was 5.3 minutes, compared to 8 or 9 minutes for the one-success and two-success policies. But a much longer total training time, on the order of hours, was required to get this two or three minute improvement at the final level. Because of this relative inefficiency, the Wollmer model would not prescribe such extensive additional practice.

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APPENDIX 1

Tables 3 through 8 present the basic success-failure data from TRIG and K-Laws subjects. These data were cross-tabulated so as to indicate the correlation between pretest-question responses and learning performance. As an example, from Table 5, it appears that there were seven persons who answered "Yes" to pretest question 6 and succeeded in doing their level-6 learning without a failure; Table 7 reveals that there were only two persons who answered "Yes" to question 6, and then failed at least once when they actually attempted the level 6 material. As a rough estimate, then, the probability is something like 7/9 that, if a person answers "Yes" to pretest item 6, he will go through the level 6 teaching without a mistake. Wollmer's model and estimation procedures are designed to take advantage of such contingencies.

Estimate of \bar{X}_{1i} and \bar{X}_{2i} are given in Tables 9 and 10 for the TRIG instruction and in Tables 11 and 12 for the Kirchhoff's Laws course.

Tables 13 and 14 show the mean times on time spent at each level by students. For the K-Laws course, the computer system recorded the time in minutes with 1 second for less than one minute, 61 seconds for between one and two minutes, and so forth. To compensate for this recording circumstance, 29 seconds were added to all times shown in Table 14.

Note that Tables 9 through 12 give the proportion of failures for TRIG and K-Laws based on the number of successes at the preceding level. However, at level 5 for TRIG (11 for K-Laws) there is no preceding level. For these levels, all data are grouped under the 1 success tables. Thus Table 10 (12) has no entry for level 5 (11).

Table 3

The Number of Successes by TRIG Students Who Answered
Yes to the Pretest Question j (W Matrix) and No
to the Pretest Question j (X Matrix)

		W Matrix (YES)					X Matrix (NO)				
(J)		LEVELS (i)					LEVELS (i)				
		1	2	3	4	5	1	2	3	4	5
	5	15	25	22	33	47	20	40	35	44	90
	4	8	13	11	12	29	27	52	46	65	108
	3	16	27	23	25	45	19	38	34	52	92
	2	14	23	20	19	40	21	42	37	58	97
	1	11	12	15	14	28	24	53	42	63	109
		1 correct solution at level i+1					1 correct solution at level i+1				
	5	18	23	24	17	0	24	34	50	49	0
	4	12	14	15	17	0	32	43	59	49	0
	3	14	19	21	18	0	28	48	53	48	0
	2	12	14	14	18	0	30	43	60	48	0
	1	8	14	12	12	0	34	43	62	54	0
		2 correct solutions at level i+1					2 correct solutions at level i+1				

Table 4

The Number of Failures by TRIG Students Who Answered
Yes to the Pretest Question j (Y Matrix) and No
to the Pretest Question j (Z Matrix)

		Y Matrix (YES) LEVELS (i)					Z Matrix (NO) LEVELS (i)				
(D)		1	2	3	4	5	1	2	3	4	5
		1 correct solution at level i+1	2 correct solutions at level i+1	1 correct solution at level i+1	2 correct solutions at level i+1	1 correct solution at level i+1	1 correct solution at level i+1	2 correct solutions at level i+1	1 correct solution at level i+1	2 correct solutions at level i+1	1 correct solution at level i+1
5	10	15	10	3	25	56	70	31	26	75	75
4	4	10	6	0	15	62	75	35	29	85	85
3	34	15	13	1	34	32	70	28	28	66	66
2	30	15	14	0	26	36	70	27	29	74	74
1	9	9	9	2	21	57	76	32	27	79	79
5	36	16	22	7	0	137	28	68	20	0	0
4	20	13	13	9	0	153	31	77	18	0	0
3	50	15	27	2	0	123	29	63	25	0	0
2	24	10	18	2	0	149	34	72	25	0	0
1	53	13	8	2	0	120	31	82	25	0	0

Table 5

The Number of Successes by K-Laws Students
Who Answered Yes to the Pretest Questions
(W Matrix)

		LEVEL (i)											
		1	2	3	4	5	6	7	8	9	10	11	
Pretest Question j	11	2	2	2	3	3	3	3	3	3	3	5	1 correct solution at level i + 1
	10	4	4	4	5	5	5	5	5	5	5	11	
	9	3	3	3	4	4	4	4	4	4	4	6	
	8	3	3	3	4	4	4	4	4	4	4	8	
	7	3	3	3	3	3	3	3	3	3	3	11	
	6	6	6	6	7	7	7	7	7	7	7	21	
	5	9	9	10	8	10	10	10	10	9	9	28	
	4	6	6	7	6	7	7	7	7	7	6	25	
	3	6	6	7	6	7	7	7	7	7	6	19	
	2	10	11	12	9	9	12	12	12	11	11	33	
	1	11	12	13	10	13	13	13	13	12	11	33	
	11	0	0	0	0	0	0	0	0	0	0	0	2 correct solutions at level i + 1
	10	4	4	4	4	4	4	4	4	4	4	0	
	9	0	0	0	0	0	0	0	0	0	0	0	
	8	0	0	0	0	0	0	0	0	0	0	0	
	7	6	6	6	6	6	6	6	6	6	6	0	
	6	12	10	12	10	12	12	12	12	12	12	0	
	5	16	14	16	14	16	16	16	16	16	16	0	
	4	16	14	16	14	16	16	16	16	16	16	0	
	3	10	10	10	10	10	10	10	10	10	10	0	
	2	20	18	20	18	20	20	20	20	20	20	0	
	1	18	16	18	16	18	18	18	18	18	18	0	

Table 6

The Number of Successes by K-Laws Students
Who Answered No to the Pretest Questions
(X Matrix)

		LEVEL (1)											
		1	2	3	4	5	6	7	8	9	10	11	
Pretest Question j	11	11	11	12	9	12	12	12	12	11	10	40	1 correct solution at level i + 1
	10	9	9	10	7	10	10	10	10	9	9	34	
	9	10	10	11	8	11	11	11	11	10	10	39	
	8	10	10	11	8	11	11	11	11	10	9	37	
	7	10	10	11	9	12	12	12	12	11	10	34	
	6	7	8	8	5	8	8	8	8	7	6	24	
	5	4	5	5	4	5	5	5	5	5	4	17	
	4	7	7	8	7	8	8	8	8	7	7	20	
	5	7	7	8	6	8	8	8	8	7	7	26	
	3	4	4	4	3	4	4	4	4	3	3	12	
	1	1	1	1	1	1	1	1	1	1	1	1	
	11	13	13	13	13	14	14	14	14	14	14	0	2 correct solutions at level i + 1
	10	11	11	11	10	12	12	12	12	12	12	0	
	9	13	13	13	12	14	14	14	14	14	14	0	
	8	12	12	12	11	13	13	13	13	13	13	0	
	7	10	10	10	9	11	11	11	11	11	11	0	
	6	7	7	7	7	8	8	8	8	8	8	0	
	5	5	5	5	5	6	6	6	6	6	6	0	
	4	5	5	5	5	6	6	6	6	6	6	0	
	3	8	8	8	7	9	9	9	9	9	9	0	
	2	3	3	3	3	4	4	4	4	4	4	0	
	1	4	4	4	4	5	5	5	5	5	5	0	

Table 7

The Number of Failures by K-Laws Students
Who Answered Yes to the Pretest Questions
(Y Matrix)

		LEVEL (i)										
		1	2	3	4	5	6	7	8	9	10	11
Pretest Question j	11	1	0	0	2	0	0	2	0	0	0	0
	10	1	0	0	2	0	0	2	0	0	0	0
	9	0	0	0	1	0	1	2	0	0	0	0
	8	1	0	0	2	0	1	2	0	0	0	0
	7	1	0	0	1	0	1	2	0	0	0	0
	6	1	0	0	2	0	2	2	0	0	0	0
	5	1	0	2	2	0	1	5	2	0	0	1
	4	0	0	2	1	0	1	5	2	0	0	0
	3	1	0	2	2	0	2	5	1	0	0	0
	2	1	0	2	2	0	2	5	2	0	0	0
	1	8	3	2	3	4	2	6	3	0	0	1
	11	0	0	0	0	0	0	0	0	0	0	0
	10	0	3	0	0	0	0	2	3	0	0	0
	9	0	0	0	0	0	0	0	0	0	0	0
	8	0	0	0	0	0	0	0	0	0	0	0
	7	0	0	0	0	5	0	1	1	0	1	0
	6	0	0	0	0	2	1	4	3	0	1	0
	5	0	3	0	0	5	1	6	8	0	1	0
	4	0	3	0	0	5	1	6	8	0	1	0
	3	0	3	0	0	3	1	3	5	0	0	0
	2	0	3	0	0	5	1	6	8	0	0	0
	1	0	3	0	0	5	1	6	8	0	0	0
1 correct solution at level i + 1												
2 correct solutions at level i + 1												

Table 8

The Number of Failures by K-Law Students
Who Answered No to the Pretest Questions
(Z Matrix)

		LEVEL (i)										
		1	2	3	4	5	6	7	8	9	10	11
Pretest Question j	11	7	3	2	1	4	2	4	4	1	0	1
	10	7	3	0	1	4	2	1	3	1	0	1
	9	8	3	0	2	4	1	1	3	1	0	1
	8	7	3	2	1	4	1	4	4	1	0	0
	7	7	3	2	2	4	1	4	4	1	0	3
	6	7	3	2	1	4	0	4	4	1	0	3
	5	7	3	0	1	4	1	1	2	1	0	2
	4	8	3	0	2	4	1	1	2	1	0	3
	3	7	3	0	1	4	0	1	3	1	0	2
	2	7	3	0	1	4	0	1	0	1	0	3
	1	0	0	0	0	0	0	0	0	0	0	2
	11	0	3	0	0	5	1	6	8	0	2	0
	10	0	0	0	0	5	1	3	5	0	2	0
	9	0	3	0	0	5	1	6	8	0	2	0
	8	0	3	0	0	5	1	6	8	0	2	0
	7	0	3	0	0	0	1	7	7	0	1	0
	6	0	3	0	0	3	0	4	5	0	1	0
	5	0	0	0	0	0	0	2	0	0	1	0
	4	0	0	0	0	0	0	2	0	0	1	0
	3	0	0	0	0	2	0	4	3	0	2	0
	2	0	0	0	0	0	0	2	0	0	1	0
	1	0	0	0	0	0	0	2	0	0	1	0
												1 correct solution at level i + 1
												2 correct solutions at level i + 1

Table 9

Proportion of Unsuccessful Attempts in TRIG Course
Following 1 Correct Solution at the Previous Level
(Question j Corresponds to Level i)

		PRETEST QUESTION NUMBER (j)					
		1	2	3	4	5	
Level (i)	5	.429	.394	.430	.341	.347	Yes answer to Pretest Question
	4	.125	.0	.038	.0	.083	
	3	.375	.412	.361	.353	.313	
	2	.429	.395	.357	.435	.375	
	1	.450	.682	.680	.333	.400	
	5	.420	.433	.418	.440	.455	No answer to Pretest Question
	4	.300	.333	.350	.309	.371	
	3	.432	.422	.452	.432	.470	
	2	.589	.625	.648	.591	.636	
	1	.704	.632	.627	.679	.737	

Table 10

Proportion of Unsuccessful Attempts in TRIG Course
Following 2 Correct Solutions at the Previous Level
(Question j Corresponds to Level i)

		PRÊTEST QUESTION NUMBER (j)					
		1	2	3	4	5	
Level (i)	4	.143	.100	.100	.346	.292	Yes answer to Pretest Question
	3	.400	.563	.563	.464	.478	
	2	.481	.417	.441	.481	.410	
	1	.869	.667	.781	.667	.667	
	4	.316	.342	.342	.269	.290	No answer to Pretest Question
	3	.569	.545	.543	.566	.576	
	2	.419	.442	.433	.419	.452	
	1	.779	.832	.815	.826	.851	

Table 11

Proportion of Unsuccessful Attempts in Kirchhoff Laws Course
Following 1 Correct Solution at the Previous Level
(In this Table, Question j is Directed to Level 12-i)

		PRETEST QUESTION NUMBER (j)											
		11	10	9	8	7	6	5	4	3	2	1	
Level (i)	1	.333	.200	0	.250	.250	.143	.100	0	.143	.091	.421	Yes answer to pretest question
	2	0	0	0	0	0	0	0	0	0	0	.200	
	3	0	0	0	0	0	0	.167	.222	.222	.143	.133	
	4	.400	.286	.200	.333	.250	.222	.200	.143	.250	.182	.231	
	5	0	0	0	0	0	0	0	0	0	0	.236	
	6	0	0	.200	.200	.250	.222	.091	.125	.222	.143	.133	
	7	.400	.286	.333	.333	.400	.222	.333	.417	.417	.294	.316	
	8	0	0	0	0	0	0	.167	.222	.125	.143	.188	
	9	0	0	0	0	0	0	0	0	0	0	0	
	10	0	0	0	0	0	0	0	0	0	0	0	
	11	0	0	0	0	0	0	.034	0	0	0	.029	
	1	.389	.438	.444	.412	.412	.500	.636	.533	.500	.636	0	No answer to pretest question
	2	.214	.250	.231	.231	.231	.273	.375	.300	.300	.429	0	
	3	.143	0	0	.154	.154	.200	0	0	0	0	0	
	4	.100	.125	.200	.111	.182	.167	.200	.222	.143	.250	0	
	5	.250	.286	.267	.267	.250	.333	.444	.333	.333	.500	0	
	6	.143	.167	.083	.083	.077	0	.167	.110	0	0	0	
	7	.250	.091	.083	.267	.250	.333	.167	.111	.111	.200	0	
	8	.250	.231	.214	.267	.250	.333	.286	.270	.273	0	0	
	9	.083	.100	.091	.091	.083	.125	.167	.125	.125	.250	0	
	10	0	0	0	0	0	0	0	0	0	0	0	
	11	0	0	0	0	0	0	0	0	0	0	0	

Table 12

Proportion of Unsuccessful Attempts in Kirchhoff Laws Course
Following 2 Correct Solutions at the Previous Level
(In this Table, Question j is directed to Level 12-i)

		PRETEST QUESTION NUMBER (j)											
		11	10	9	8	7	6	5	4	3	2	1	
Level (i)	1	0	0	0	0	0	0	0	0	0	0	0	Yes answer to pretest question
	2	0	.429	0	0	0	0	.176	.176	.231	.143	.157	
	3	0	0	0	0	0	0	0	0	0	0	0	
	4	0	0	0	0	0	0	0	0	0	0	0	
	5	0	0	0	0	.455	.143	.238	.238	.231	.200	.217	
	6	0	0	0	0	0	.077	.059	.059	.091	.048	.053	
	7	0	.33	0	0	.143	.250	.273	.273	.231	.231	.250	
	8	0	.429	0	0	.143	.200	.333	.333	.333	.286	.308	
	9	0	0	0	0	0	0	0	0	0	0	0	
	10	0	0	0	0	.143	.077	.059	.059	0	0	0	
	1	0	0	0	0	0	0	0	0	0	0	0	No answer to pretest question
	2	.188	0	.188	.200	.231	.300	0	0	0	0	0	
	3	0	0	0	0	0	0	0	0	0	0	0	
	4	0	0	0	0	0	0	0	0	0	0	0	
	5	.263	.294	.263	.278	0	.273	0	0	.182	0	0	
	6	.067	.077	.067	.071	.083	0	0	0	0	0	0	
	7	.300	.200	.300	.316	.389	.333	.250	.250	.308	.333	.286	
	8	.364	.294	.364	.381	.389	.385	0	0	.250	0	0	
	9	.083	.100	.091	.091	.083	.125	.167	.125	.125	.250	0	
	10	.125	.143	.125	.133	.083	.111	.143	.143	.182	.200	.167	

Table 13

Time Data for the TRIG Course (N = 80)

level	Total Time (Secs)	Number of Problems	Average Time (Min.) per Problem
1	76830	316	4.05
2	81788	251	5.43
3	82599	262	5.25
4	19620	199	1.65
5	29015	237	2.03

Table 14

Time Data for the K-Laws Course (N = 30)

Level	Total Time (Secs)	Number of Problems	Average Time (Min.) per Problem
1	40771	72	9.91
2	16754	62	4.98
3	25774	71	6.53
4	21755	62	6.33
5	29483	76	6.95
6	8455	54	3.10
7	6870	60	2.40
8	9031	61	2.95
9	1728	48	1.08
10	1185	45	.92
11	1244	44	.95